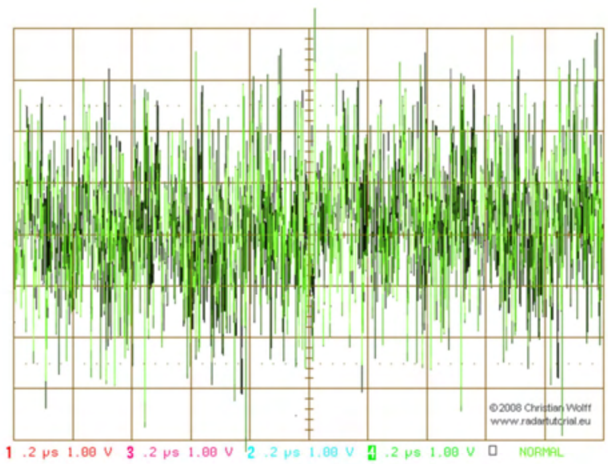
Arraying Radiometers For Realizing Synthetic Apertures

IEEE Synthetic Aperture Radiometry Study Group

November 17, 2022

In our kick-off meeting in July, an overview acquainted us with radiometry. At the next meeting we delved into the general characteristics of instruments employed in radiometry, namely radiometers. This exposure focuses on arraying radiometers for aperture synthesis.

Temporal Behavior of Noise



Courtesy: radartutorial.ecu

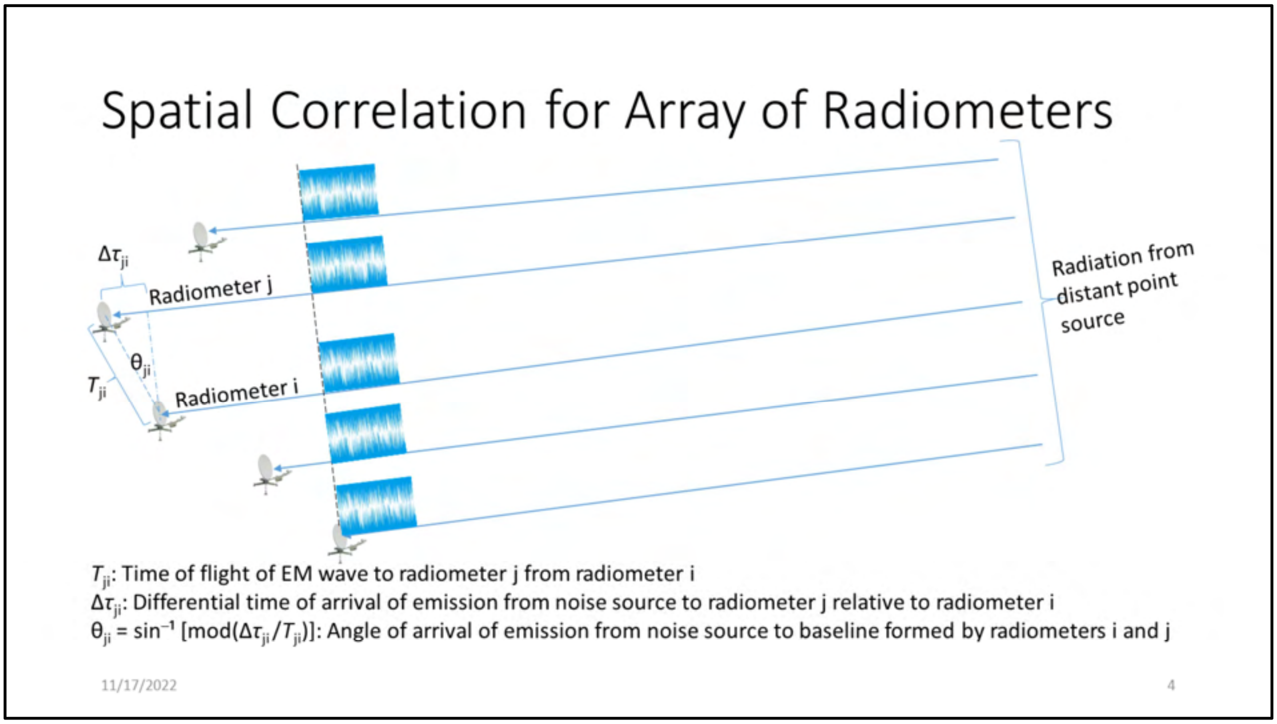
The aperiodic and a priori unknown behavior of emitted noise imposes different considerations on realization of synthetic apertures for radiometer arrays compared to synthetic arrays for radar. Let us recap some of the important properties of random noise to understand these differences.

Temporal Behavior of Random Noise

- A complex random voltage $v_c(t) = v_{Re}(t) + jv_{Im}(t)$ from a black body:
  - Has statistically independent $v_{Re}(t)$ and $v_{Im}(t)$, that has zero mean after band-pass filtering by a radiometer, and conforms to zero-mean Gaussian distributions in the interval $[-\infty < v_{Re}(t) < \infty, -\infty < v_{Im}(t) < \infty]$ with identical variances.
    - $\overline{v_{Re}(t)v_{Re}(t+\tau)} = \overline{v_{Im}(t)v_{Im}(t+\tau)} = 0$ for all $\tau \neq 0$
  - The distribution for $|v_c(t)|$ is Rayleigh over $[0 \leq |v_c(t)| < \infty]$.
    - Mean of the distribution equals its variance
    - $\overline{|v_c(t)||v_c(t+\tau)|} = 0$ for all $\tau \neq 0$
  - The distribution for $\arg[v_c(t)]$ is uniform over $[-\pi \leq \arg[v_c(t)] \leq \pi]$, and zero otherwise.
    - Zero mean and variance = $\frac{1}{3}\pi^2$.
    - $\overline{\arg[v_c(t)]\arg[v_c(t+\tau)]} = 0$ for all $\tau \neq 0$
- No temporal coherence of voltage in any of its statistical manifestations provided that averaging time is long enough.

Recall at our previous meeting that I pointed out that band-pass filtering eliminates the mean of the voltage fluctuations and retains only deviations from that mean. Thus, at the band-pass filter's output, the complex voltage consists of statistically independent real and imaginary parts that are zero-mean Gaussian distributions with identical variances. The voltage at any instant is uncorrelated with the voltage at any other different instant. This applies to transformations of the same voltage whether viewed in real-imaginary or magnitude-phase formats. There is no temporal correlation among time-sequenced samples of this voltage provided that the duration of the correlation process is long enough.

This is different from synthetic aperture processes developed for periodic voltage sequences, where coherence exists between time-sequenced samples of the scene because of the periodic nature of the waveform. It is also different for synthetic aperture radar that uses noise waveforms because the transmit noise sequence is known and a replica of it is available to correlate with received echoes. In synthetic aperture radiometry we must rely on spatial rather than temporal coherence between samples taken at spatially-separate radiometers.



This diagram shows a general array of radiometer receivers that have radiation emitted by a distant source incident on the array. At the spatial location denoted by the black dashed line, the noise from the distant source to radiometers at this location is correlated, so that the cross-correlation between any pair of radiometers located at that line yields the noise power from the source. However, all radiometers are not located at the dashed line; some are beyond the dashed line at different distances from it. Thus, the radiation arrives at these radiometers at different times. Cross-correlation between any pair of radiometers, i and j, yields the differential time of arrival, $\Delta\tau_{ji}$, between those pairs. Denoting the time of flight of a plane wave between the two radiometers as T_{ji},, which may be determined precisely from their known positions, we obtain the angle of arrival relative to the normal to the baseline of that pair from the formula shown.

We may understand the benefit of extracting angle from differential delay from the following physical interpretation: $\Delta\tau_{ji}$ is the time delay that must be algebraically added to radiometer i, so that the times of arrival to radiometers i and j are equal. We know from basic interferometry, that the equal delay condition is achromatic. So it encompasses the entire spectrum of the received noise and not just noise at the center frequency.

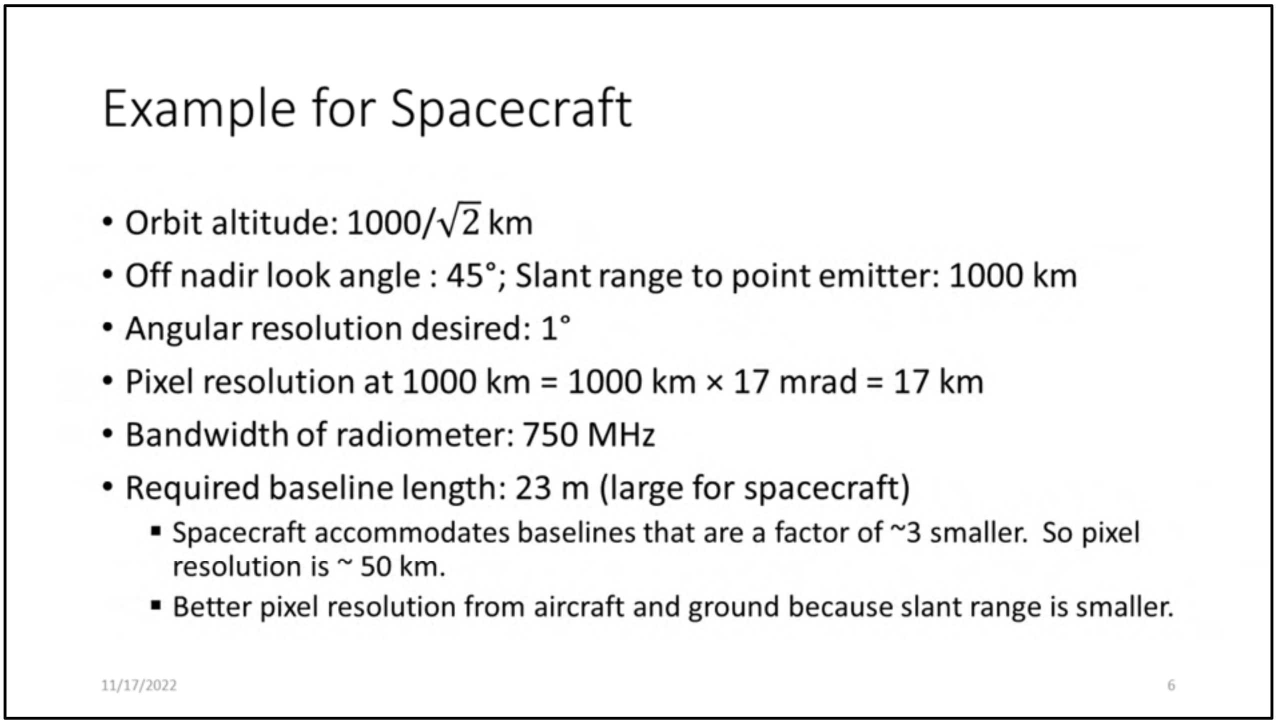
There is a more complicated relationship between differential delay and angle if the baselines are comparable in length to distance from the source. In that case, a formalism based on spherical wave-front should be adopted. This is an area to explore when we are working on the recommended practice.

Observations

- Minimum of 3 baselines, with none in the plane of the other two, to uniquely determine 3-D position of source.
  - Considerations of dilution of precision apply (similar to GDoP and PDoP for satellite-based navigation systems).
  - Ambiguity resolution (mirages) requires more than 3 baselines.
- Minimum resolvable differential delay, $\Delta\tau_{ji} > \Delta\tau = 1/B$, where B is bandwidth of radiometer.
  - Angular resolution, $\Delta\theta = \sin^{-1}$ [mod{$1/(BT_{ji})$}]
  - High resolution requires high bandwidth (limited by interference considerations for practical receivers) https://www.ncbi.nlm.nih.gov/pmc/articles/PMC8244653 and large baselines (limited by real estate for ground, air, and space.)

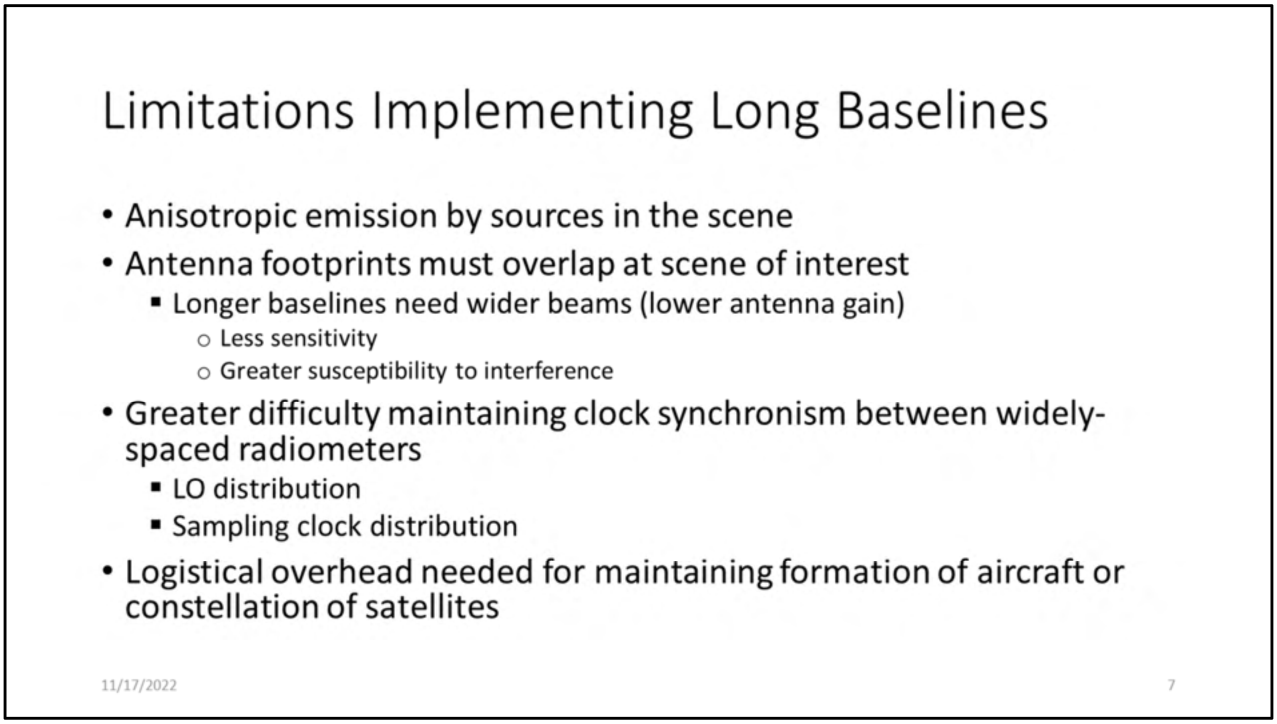
Several observations directly emerge from the description on the previous slide. A minimum of 3 baselines are needed to locate the position of a source unambiguously. Furthermore, the baselines cannot be collinear or coplanar. Each pair must define a plane that does not coincide with planes formed by other pairs. The relative positions of the vertices formed by the baselines can create ill-conditions for location determination much like Geometric Dilution of Precision (GDoP), and Position Dilution of Precision (PDoP) does for satellite based navigation systems as well as ground-based triangulation systems. In some cases, ambiguity resolution may require additional baselines.

The minimum resolvable angle is related to the bandwidth of the radiometers and the baseline distances between them. High resolution requires radiometers with large bandwidths. Practical considerations limit receiver bandwidth to what is achievable and what the platform on which it is mounted can accommodate. An approach to using bandwidth is described in the accompanying link. Large baselines are likewise limited by what the platform can accommodate as an example on the next slide shows.



In this slide we carry a calculation for a radiometer array aboard a satellite, where we desire an angular resolution of 1° or approximately 17 milliradians. We choose a radiometer that has a bandwidth of 750 MHz, which is near the best that the current state-of-the-art can offer. It is at once seen that the large slant range yields a pixel resolution of 17 km on the ground and requires a baseline of 23 m on the spacecraft. Although this is technically feasible, it requires great expense because of deployment and operation. At current state-of-the-art, arrays are about a factor of 3 smaller, leading to pixel resolutions of about 50 km from space-borne platforms.

Better resolutions are available from ground and airborne platforms because of the much smaller slant ranges.

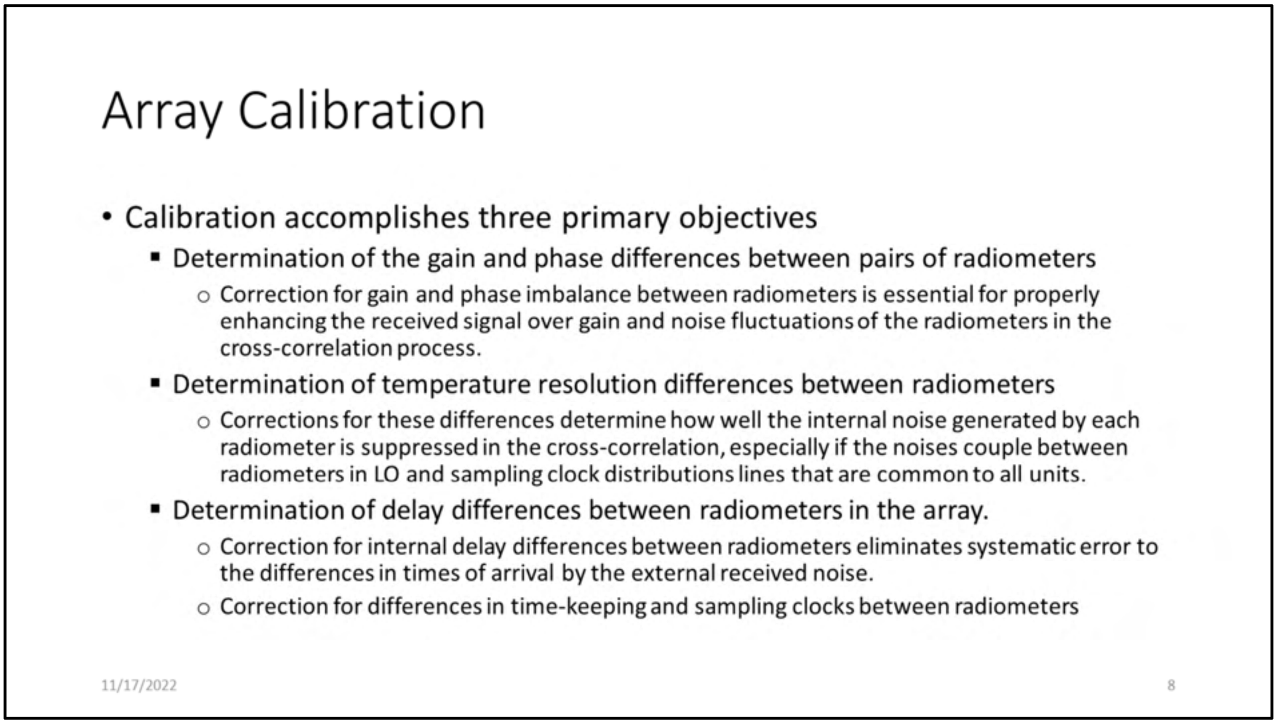


There are limitations on implementation of long baselines. If emitters in the scene are anisotropic, correlation is impossible. This is especially true of polarized emitters.

When spacing is large, the need for antenna beams to overlap at the scene of interest drives design to wider beamwidth, and therefore lower gain antennas. The lower gain reduces sensitivity of the radiometer; the wider beamwidth increases susceptibility to interference.

There is greater difficulty in maintaining synchronism between clocks on widely-separated platforms. Moreover, it is not feasible to drive radiometers on widely-spaced platforms from a common LO or sampling clock. Correlation degrades as a consequence of frequency and time mismatch.

There is logistical overhead for flying a formation of aircraft. For space applications, there is the logistics of launching multiple spacecraft and maintaining the constellation after launch.



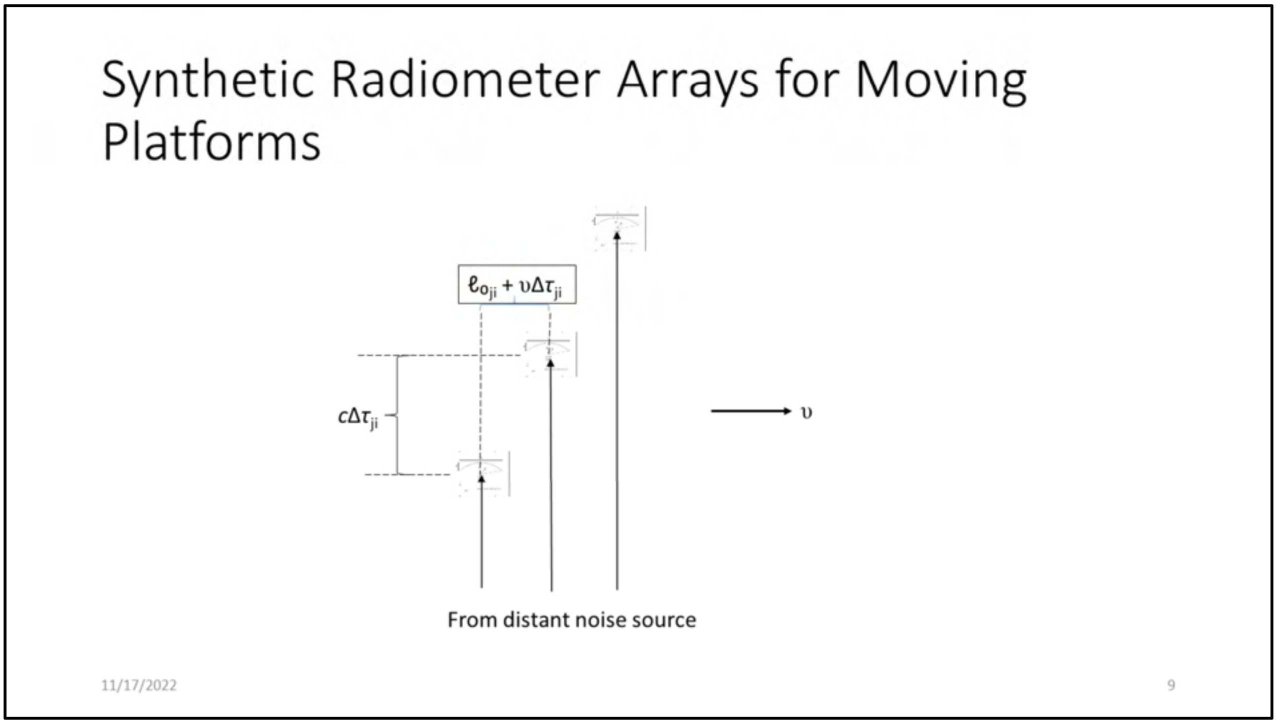
As might be expected, calibration of the array is inevitable to correct for the radiometers not being identical and having variations in their gain, phase, delay and noise characteristics.

Correction for gain and phase imbalance between radiometers is essential for properly enhancing the received signal over gain and noise fluctuations of the radiometers in the cross-correlation process. This is one instance where relative phase is meaningful in radiometry.

Corrections for noise characteristics determine how well the internal noise generated by each radiometer is suppressed in the cross-correlation, especially if the noises couple between radiometers in LO and sampling clock distributions lines that may be common to all units.

Correction for internal delay differences between radiometers eliminates systematic error from the differences in times of arrival by the external received noise. Part of this is contributed from differences in propagation delays through the radiometers and part of this is due to imperfect synchronization of time-keeping clocks and sampling clocks.

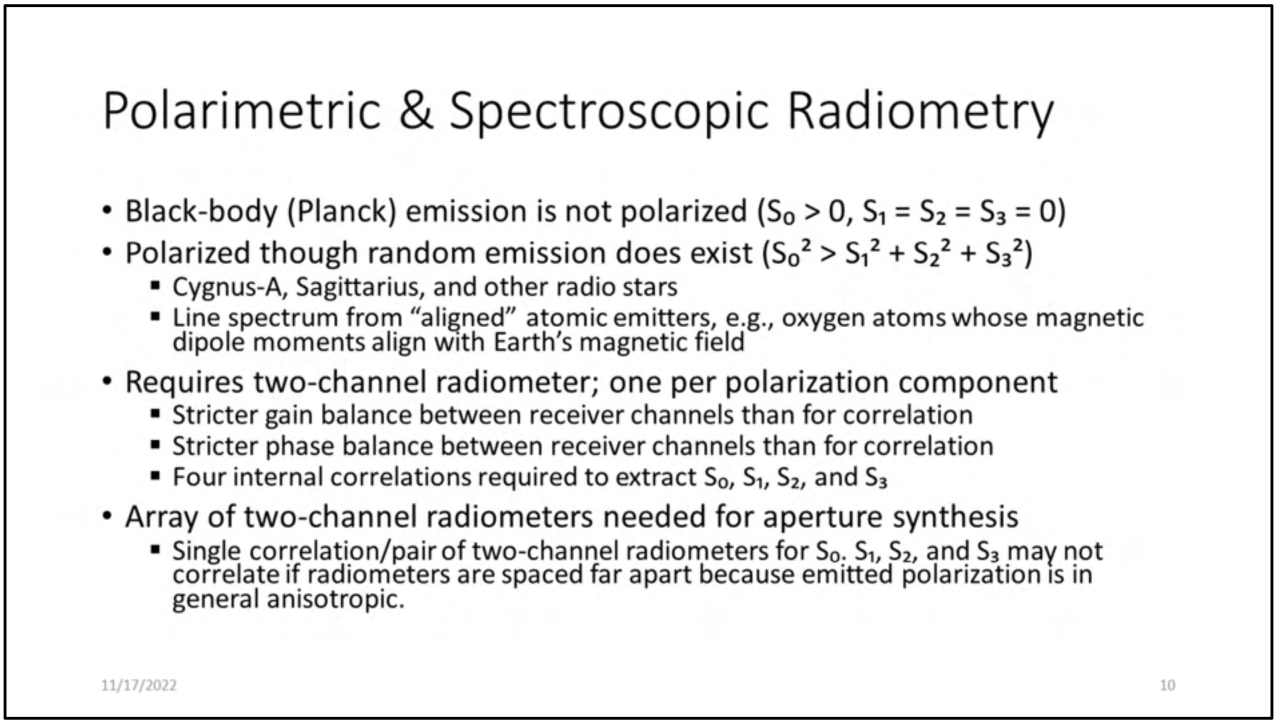
I anticipate that members will make significant contributions to the standard in the arena of calibration methods.



To synthesize a moving array, consider that the arrangement of radiometers may be displaced by an amount $c\Delta\tau_{ji}$ in the cross-track direction in addition to a static amount, ℓ_{oji}, in the along track direction. Cross-correlation between adjacent elements delivers delay $\Delta\tau_{ji}$. In that delay time, the trailing element has displaced by an amount $\upsilon\Delta\tau_{ji}$ in the along-track direction. All elements receive the same voltages albeit at different delay times, $\Delta\tau_{ji}$. Their displacement in the along-track direction enables construction of a synthetic aperture, where the dimensionless ratio, υ/c, replaces phase displacement between adjacent elements.

Evidently, each point source will present different Doppler shifts to different elements in the array. For example, if the line of sight of the central element in the diagram is aligned with the point source and orthogonal to the platform motion, that element will experience no Doppler shift. However, the leading element will experience receding Doppler, whereas the trailing element will experience approaching Doppler. Thus, the fluctuations at the trailing element will appear to be more rapid than the central element, whereas the fluctuations experienced by the leading element will appear to be less rapid. In effect, Doppler variation among elements will degrade the correlation depending on the motion of the platform and the desired resolution. Moreover, the variations in Doppler varies with position of the array relative to the point source. When all elements are left of the source, they all experience positive Doppler although by varying amounts. Likewise, when all elements have passed by the source they experience negative Doppler. Clearly, cross correlation between elements must be preceded by compensation of this varying Doppler.

We could include this scheme for synthetic aperture radiometry for moving platforms if members concur.



Although many references in the literature provide details about polarization and polarimetric radiometry, a few points are worth noting. First, the concept of polarization by definition requires its observation over a finite non-zero duration. Polarization is NOT an instantaneous concept. Black body radiation as exemplified by Planck’s law is not polarized as confirmed by measured components of the Stokes’ vector.

Nevertheless, random noise sources that emit polarized electromagnetic radiation do exist as in radio stars; Cygnus-A and Sagittarius for example. Certain atomic species that possess a dipole moment emit polarized noise when those dipole moments are aligned by an external field. For example, oxygen atoms possess a magnetic dipole moment and an ensemble of these magnetic moments aligned by Earth’s magnetic field emit partially polarized radiation in a line spectrum that peaks at 118.75 GHz. Line spectrum emitters are not black-body emitters whose spectrum is flat over the measurement band.

A polarimetric radiometer with spectroscopic capability is required to characterize emission from polarized line sources. Such radiometers are two-channel receivers; one for each polarization component which may be either horizontal-vertical (H, V), or left-hand- right hand-circular (LHC, RHC). Because polarization state involves both amplitude and relative phase between orthogonal basis polarizations, it is essential that the relative gains and relative phase between the two receiver channels in a radiometer be closely matched. This is another instance where phase is meaningful for radiometry. It takes four correlations within each radiometer element to extract all four components of the Stokes vector.

Aperture synthesis with polarimetric elements increases the number of correlations by a factor of 4 over non-polarimetric aperture synthesis. However, such correlations require care if the source polarization is anisotropic.

Limitations

- Temperature is attributed to a system in thermodynamic equilibrium which means that the system has
  - No net force acting on it
  - No gradients (temperature or concentration)
  - No net energy transfer between one part of the system and another
  - Moments of governing statistical distribution do not vary with time (in practice moments vary over time durations that are large compared with observation time)
- Highly non-equilibrium scenes, e.g., a raging forest fire, may not be accessible to actionable observation

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There is a fundamental limitation that arises out of the assignment of temperature or distribution of temperature to a scene. The attribution of noise power to a temperature applies to systems in thermodynamic equilibrium, i.e., the system has no net force acting on it and has neither temperature nor concentration gradients, nor any net energy transfer between its parts. Under these conditions, the moments of the statistical distribution that governs its random behavior are stationary in time.

This concept is extended to steady state non-equilibrium situations where a spatial temperature distribution may exist but it is stationary in time, e.g., a thermally conducting bar constantly heated at one end. In this case, the statistical moments of the distribution depend on position but are stationary in time. The concept is further extended to quasi-stationary situations where the time duration over which the statistical moments vary is long compared to the observation time.

As we have discussed, synthetic aperture radiometry entails a minimum averaging time so that uncorrelated components of noise may be adequately suppressed relative to correlated noise from the scene. If however, temporal variations of noise power from the scene have shorter duration than the needed correlation time, such scenes may not be accessible to radiometric observation that is sufficiently meaningful to be actionable.

Alternative to Correlation (tapestry approach)

- Uncorrelated array of radiometers https://www.eoportal.org/satellite-missions/tempest-d#mm-radiometer-millimeter-wave-radiometer
  - Each narrow-beam radiometer measures a part of the scene
  - Output from all radiometers stitched together to complete the tapestry
  - Spatial resolution determined by antenna beamwidth

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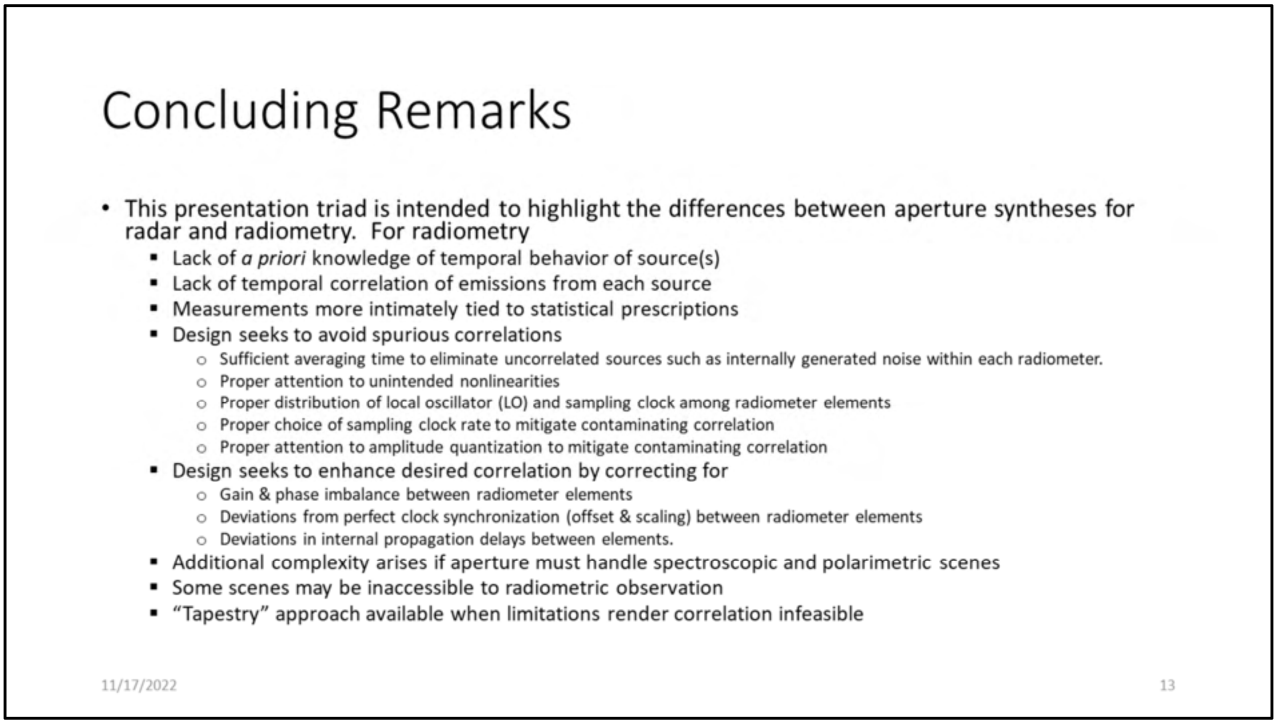
So the question arises as to what can be done when correlation cannot be implemented. A "tapestry" approach provides an answer as was implemented with TEMPEST-D and described in the accompanying link. This approach uses an uncorrelated array of radiometers. Each radiometer independently performs measurement within its field of view. The measurements from all radiometers are stitched together to render an image of the scene. Spatial resolution is determined by the beamwidth of the antenna at each radiometer. There is no synthetic enhancement of resolution.



Three presentations over four meetings of the study group were intended to highlight differences in the way aperture synthesis is realized in radar and radiometry. Differences arise because the radiometer has no reference source against which to compare its received signal. Furthermore, the signal from sources in radiometry is noise for which temporal correlation does not exist and so processing is tied more intimately to statistical prescription for random temporal behavior. Aperture synthesis in radiometry must appeal to a different set of considerations than for radar.

The central consideration in radiometry is to suppress spurious correlations that would otherwise contaminate the correlations from the scene. A major thrust in this direction is allowance of sufficient correlation time so that uncorrelated noise has time to decorrelate. This is because the amount of suppression of uncorrelated noise is directly related to the correlation time allowed. This is the reason why low-noise radiometers perform better than ones with poorer noise performance; suppression of noise from the former consumes less correlation time to reach a certain level, usually the confusion limit of the radiometer.

Another thrust is to avoid introducing correlation into the input where none exists. Such correlations can occur in distribution of common sources such as LOs and sampling clocks as well as unintended nonlinearities in each radiometers. Non-linearity has the effect of grafting internally-generated noise of the radiometer on to external noise from the scene.

Amplitude quantization produces correlation of an otherwise uncorrelated sequence if the quantization step is comparable with the standard deviation of the input noise. Likewise, temporal quantization produces correlation because of the response time of the anti-aliasing filter and its relation to the sampling rate. Unlike radar where some benefit accrues from over-sampling, in radiometry, correlation between samples gets worse if the input noise is over-sampled.

On the flip side of suppression, radiometer design seeks to enhance the correlation of noise from the scene by calibration that corrects for gain and phase imbalance between radiometer elements, deviations of time-keeping and sampling clocks from perfect synchronization, and deviations in internal propagating delays between radiometers. The complexity of determining these corrections and implementing them increases if the scene to be characterized requires polarimetric and/or spectroscopic handling.

A number of these considerations already establish a collection of recommended practices, but do not address calibration methods that support how the many corrections will be determined. This is one such effort that the working group can undertake. There are others that have been sprinkled throughout this presentation triad, and I hope that these presentations supply the basis for constructing a strong document of recommended practices for aperture synthesis in radiometry.

Transition to Working Group

- Formality markedly increases at meetings (quorum, agenda, minutes)
- Contributions to recommended practice comes from members
  - No contributions from chair (prevent perception of bias and preserve neutrality)
- Call for patents and call for copyrights by chair at start of each meeting
- Acceptance of contribution(s) by formal vote by members
  - No vote by chair except in special circumstances (making/breaking tie)
  - Goal is to accept by 75% of vote
  - Only members of IEEE and IEEE-SA are entitled to vote
- Writing of draft recommended practice shall follow IEEE-SA style guide

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With the transmittal of PAR #P3339 to NESCOM, the study group will be on hiatus until approval to form a working group.

Working group proceedings are more formal than study group. Meeting must meet quorum requirements of voting members. Agenda & minutes and approvals of them are more formal.

An important change is contributions, which must come from members (voting and non-voting) and not from the chair. This is to prevent perception of bias or dominance and to ensure that the chair is neutral in all matters relating to contributions.

The chair will issue separate calls for patents and copyrights pertaining to all offered contributions. Chair will provide form letters to contributors to fill out regarding the intent and scope of use of patent or copyright material in the recommended practice.

Acceptance of contributions into the recommended practice is by vote by all eligible members. The chair does not vote except in special circumstances: to break a tie or make a tie. The goal is that each contribution gains approval by 75% of the vote.

Once a contribution is accepted, it is incorporated into the draft in several places as specified in the style guide. Members are urged to consult the style guide both for adherence to the style and to become knowledgeable about the review process.