DR-BrianSequeira-01
Y-Factor Method

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Purpose of DR

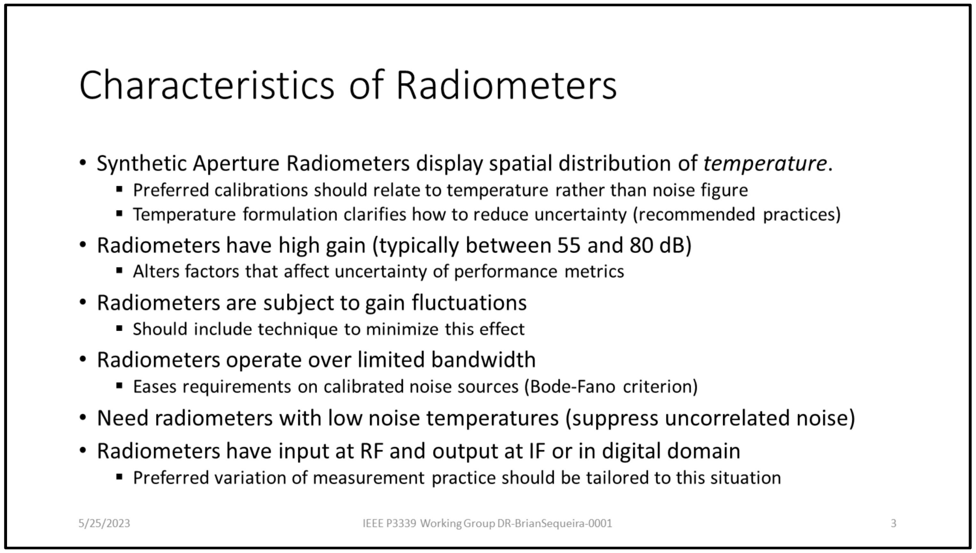
- Report conflict between formula in document that is widely referenced on the world wide web and finding by author (Sequeira) https://www.keysight.com/us/en/assets/7018-06808/application-notes/5952-8255.pdf
- Present alternate analysis that clarifies rationale for selection of best practices for radiometers. https://www.keysight.com/us/en/assets/7018-06829/application-notes/5952-3706.pdf
- Stimulate awareness on consistent notation among contributors.
- Assess effectiveness of DR process.

There is a discrepancy between formula for noise factor derived here and that reported in a document that is widely distributed on the world-wide web. The primary objective of this report is to bring this discrepancy to the attention of the working group for adjudication and disposition in relation to the Recommended Practices document.

Much of the available literature addresses noise characterization of low-noise amplifiers (LNAs) but it is relatively sparse when addressing noise performance of radiometers. A secondary objective of this report is to address that gap.

A third objective is to stimulate some thought about a consistent notation convention used by all contributors, so that, for example, capital T does not represent time in one contribution and absolute temperature in another.

Finally, this DR provides a test case for the effectiveness of the DR process.



In general purpose research, noise performance is characterized by noise factor, or its logarithmic equivalent, noise figure. A synthetic aperture radiometer array on the other hand, maps the spatial distribution of temperature over the scene. Thus, calibration that puts temperature at the forefront, more directly connects with the image product created by processing the array. As will be seen, an analysis that is temperature-centric not only achieves that end but also provides a direct rationale for improving calibration outcomes. It is integral to our case for recommending best practices.

Radiometers have high gain; a fact that diminishes the significance of some sources of error that are important in general-purpose situations. However, radiometers are susceptible to gain fluctuations which general purpose applications do not consider. So calibration of radiometers should include techniques for mitigating this as we will see shortly.

Radiometers operate over limited bandwidth, and therefore need not use wideband noise sources that are designed for general-purpose use. Instead, noise sources can be designed for bandwidths that are compatible with the radiometer. This is advantageous because noise sources can achieve better match over smaller bandwidths, and better match yields smaller uncertainties.

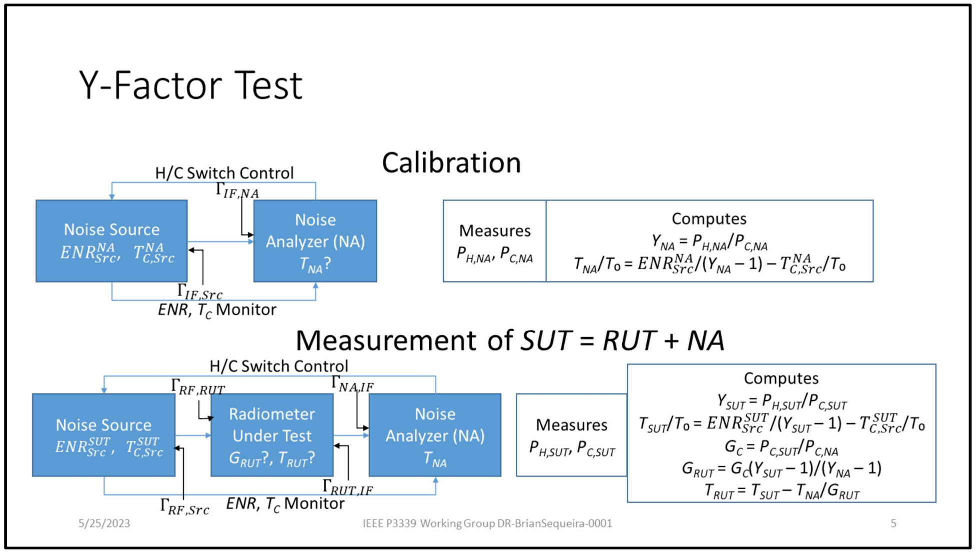
Aperture synthesis needs radiometers with low noise temperatures to suppress uncorrelated noise within limited aperture time. A radiometer with noise temperature of 435 K (NF = 3.79 dB) requires 16 times the correlation time to achieve the same decorrelation floor as one with a noise temperature of 80 K (NF = 1.06 dB). Consequently, calibration methods and devices should target low to moderate noise temperatures.

Lastly, a radiometer's operating frequency at the input, which is at RF, and output which is at IF, are different. A noise figure analyzer operates at IF, and its noise characteristics are different from that of the radiometer. The noise characteristics of the calibrating noise source should match those of the unit to which it is connected to obtain optimum outcome.

Terminology

- Reference temperature for noise measurements, T_0 = 290 K
- Noise temperature referred to the input of Set-up Under Test (SUT), T_{SUT} K
- Cold (OFF?) temperature of noise source connected to SUT, $T_{C,Src}^{SUT}$ K
- Hot (ON?) temperature of noise source connected to SUT, $T_{H,Src}^{SUT}$ K
- Measured noise power at SUT output driven by source at $T_{C,Src}^{SUT}$, $P_{C,SUT}$ W
- Measured noise power at SUT output driven by source at $T_{H,Src}^{SUT}$, $P_{H,SUT}$ W
- Y-factor of SUT, $Y_{SUT}(f) = P_{H,SUT}/P_{C,SUT}$
- Excess noise temperature of source, $ENR_{Src}^{SUT}(f) = (T_{H,Src}^{SUT} - T_{C,Src}^{SUT})/T_0$
- Set-up noise factor, $F_{SUT} = 1 + T_{SUT}/T_0$; Noise figure, $NF_{SUT} = 10 \log_{10} F_{SUT}$ dB

Here is a list of terms used in this treatment. Please consult IEEE's glossary & dictionary of terms and use them where possible. If the glossary or dictionary does not define a term, use a consistent convention for its definition. For example, I use a subscript to associate a property with a unit; T_{SUT} to denote the noise temperature of the set-up under test. If the property has an underlying condition, e.g. hot or cold, I have that condition represented as a subscript that precedes the association with the unit and separated from it by a comma. Thus, $P_{C,SUT}$ represents the power output from the set-up under test when driven by a cold source. At times a quantity must be distinguished from another according to the unit to which it is connected. In this case, I use a superscript to identify the connecting unit.



This slide summarizes the estimation of the noise temperature and gain of the radiometer under test. I have used the term noise analyzer (NA) instead of noise meter to convey that that block has a measurement role and a computational role. In keeping with the radiometer operating at RF at its input and IF at its output, the NA operates at IF as indicated by the subscript on its reflection coefficient, gamma. The NA is characterized in addition by its noise temperature, which is shown with a question mark in the upper diagram to convey that it is determined in that configuration. Noise sources are characterized by temperature under cold operation, excess noise ratio, and source mismatch, which for the upper diagram is operating at IF. Blocks to the right of the set-up describe what the NA measures and what it computes from those measurements. The NA has a control line to toggle the noise source between its states. It also monitors variations in cold temperature and *ENR* of the noise source.

The lower diagram describes the determination of the gain and noise temperature of the radiometer under test. The NA is the same as in the upper diagram, but the noise source operates at RF and has different characteristics at this range of frequencies. As seen from the blocks on the right, the determination of the characteristics of the radiometer employ measurements and computations from both upper and lower configurations.

In either case, the noise characteristics of the source are determined by and traced to a standards lab, and supplied as a software-readable table as function of frequency. As seen from the “Measures” and “Computes” blocks alongside, the two configurations provide sufficient information to determine gain and noise temperature of the radiometer. In both configurations, the primary measurements of power are used to derive a ratio of those powers, which is termed Y-factor. The *ENR* and T_C of the noise source in conjunction with the Y-factor determines the noise temperature of the corresponding configuration.

Assumptions Behind Computations

- Bandwidth (BW) relationship: $BW_{NA} < BW_{SUT} < BW_{Src}$
  - Frequency swept measurement to properly characterize frequency dependence of SUT gain and match, and source *ENR* and match.
- $P_{H,SUT} = k(T_{H,Src}^{SUT} + T_{SUT})G_{H,SUT}\, BW_{SUT}$; $P_{C,SUT} = k(T_{C,Src}^{SUT} + T_{SUT})G_{C,SUT}\, BW_{SUT}$
  - $Y_{SUT} = \left(\frac{G_{H,SUT}}{G_{C,SUT}}\right)\left(\frac{T_{H,Src}^{SUT} + T_{SUT}}{T_{C,Src}^{SUT} + T_{SUT}}\right)$
  - If, as assumed in referenced analyses, $G_{H,SUT} = G_{C,SUT}$, then $Y_{SUT} = \left(\frac{T_{H,Src}^{SUT} + T_{SUT}}{T_{C,Src}^{SUT} + T_{SUT}}\right)$
  - In radiometry, $G_{H,SUT} \neq G_{C,SUT}$ due to:
    - Temporal gain fluctuations
      - requires sufficiently rapid switching between H & C states so that gain is constant over switching cycle but that may affect T_C of sources that dissipate ohmic heat when hot but do not have short enough thermal time constant to recover cold temperature when current is turned off. Temperature monitoring of T_C required to adjust for this variation.
    - Nonlinear behavior at high drive levels (limits *ENR* of source)

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Inherent in the configurations shown is that the bandwidth of the NA is least, that of the source is greatest, and that of the radiometer is between these bounds. This is because, *ENR* and gain are not perfectly flat with frequency. There is some ripple in power transferred by the source and ripple in gain of the radiometer. Consequently, the NA sweeps its narrower band (~300 kHz) across the wider band (~100s of MHz) of the radiometer, which is driven by a noise source whose bandwidth envelopes that of the radiometer.

Gain fluctuations prevent the noise source from being leisurely toggled between hot and cold states of the noise source. Gain variations in a radiometer occurs over a time scale of 10s of ms. So the noise source must toggle at kHz rates to maintain nearly constant gain in each switching cycle. This supports a key assumption of constant gain for each determination of Y-factor. However, gain variations can occur because of incidental nonlinearity.

Rapid toggling can affect the noise temperature and *ENR* of the source if thermal time constant precludes recovery of temperature to its original state. This is why the block diagrams show one connection between NA and source that asserts switching between the source's hot and cold states. Through another connection, the NA monitors the *ENR* and T_C of the source.

Derivation of Determination of T_{SUT}

- Assuming $G_{H,SUT} = G_{C,SUT}$
  - $Y_{SUT} = \frac{T^{SUT}_{H,Src} + T_{SUT}}{T^{SUT}_{C,Src} + T_{SUT}} = \frac{\frac{T^{SUT}_{H,Src}}{T_0} + \frac{T_{SUT}}{T_0}}{\frac{T^{SUT}_{C,Src}}{T_0} + \frac{T_{SUT}}{T_0}}$
- By definition, $\frac{T^{SUT}_{H,Src}}{T_0} = ENR^{SUT}_{Src} + \frac{T^{SUT}_{C,Src}}{T_0}$, so
  - $Y_{SUT} = \frac{ENR^{SUT}_{Src} + \frac{T^{SUT}_{C,Src}}{T_0} + \frac{T_{SUT}}{T_0}}{\frac{T^{SUT}_{C,Src}}{T_0} + \frac{T_{SUT}}{T_0}} = \frac{ENR^{SUT}_{Src}}{\frac{T^{SUT}_{C,Src}}{T_0} + \frac{T_{SUT}}{T_0}} + 1 \quad \Rightarrow Y_{SUT} - 1 = \frac{ENR^{SUT}_{Src}}{\frac{T^{SUT}_{C,Src}}{T_0} + \frac{T_{SUT}}{T_0}}$

    $\Rightarrow (Y_{SUT} - 1)\left(\frac{T^{SUT}_{C,Src}}{T_0} + \frac{T_{SUT}}{T_0}\right) = ENR^{SUT}_{Src} \quad \Rightarrow \frac{T_{SUT}}{T_0} = \frac{ENR^{SUT}_{Src}}{Y_{SUT} - 1} - \frac{T^{SUT}_{C,Src}}{T_0}$
  - Noise Factor, $F_{SUT} = 1 + \frac{T_{SUT}}{T_0} = \frac{ENR^{SUT}_{Src}}{Y_{SUT} - 1} + 1 - \frac{T^{SUT}_{C,Src}}{T_0}$ (different from eq. 3.7 and 3.8 in reference)

This display shows the derivation of noise temperature of the SUT using the same starting point as in the online reference. However, the end result derived here differs from the presentation in the online reference. We will use the formula derived here to analyze uncertainties in the values determined.

Computational Uncertainty Estimate (Switching Eliminates Gain Fluctuation)

- $\frac{\delta T_{SUT}}{T_0} = \left(\frac{T_{C,Src}^{SUT}}{T_0} + \frac{T_{SUT}}{T_0}\right)\left\{\frac{\delta ENR_{Src}^{SUT}}{ENR_{Src}^{SUT}} - \delta[\ln(Y_{SUT} - 1)]\right\} - \frac{\delta T_{C,Src}^{SUT}}{T_0}$
- $\frac{\Delta T_{SUT}}{T_{SUT}} = \sqrt{\left(1 + \frac{T_{C,Src}^{SUT}}{T_{SUT}}\right)^2 \left\{\left(\frac{\Delta ENR_{Src}^{SUT}}{ENR_{Src}^{SUT}}\right)^2 + \left[\ln\left(1 + \frac{\Delta Y_{SUT}/Y_{SUT}}{1 - 1/Y_{SUT}}\right)\right]^2\right\} + \left(\frac{\Delta T_{C,Src}^{SUT}}{T_{C,Src}^{SUT}}\right)^2 \left(\frac{T_{C,Src}^{SUT}}{T_{SUT}}\right)^2}$
  - For $T_{C,SUT}$ = 77 K (liq. N_2), T_H = 1240 K, T_{SUT} = 80.003 K, Y_{SUT} = 7.35, $\frac{\Delta ENR_{Src}^{SUT}}{ENR_{Src}^{SUT}} = \frac{\Delta T_{C,Src}^{SUT}}{T_{C,Src}^{SUT}} = \frac{\Delta Y_{SUT}}{Y_{SUT}} = 0.03$, $\frac{\Delta T_{SUT}}{T_{SUT}} = 0.093$.
    - If $T_{H,Src}^{SUT}$ = 8777 K (avalanche diode), $\frac{\Delta T_{SUT}}{T_{SUT}} = 0.088$
    - If $T_{C,Src}^{SUT}$ = 4.2* K (liquid He), $\frac{\Delta T_{C,Src}^{SUT}}{T_{C,Src}^{SUT}} = 0.05$, $\frac{\Delta T_{SUT}}{T_{SUT}} = 0.046$

* Check if Boltzmann approximation of Planck's law holds

Taking differentials of the derived expression for T_{SUT} from previous slide, and combining errors on root sum of squares (RSS) basis, we obtain the expression under the radical sign. Biggest impact on uncertainty of T_{SUT} is selecting a noise source with a cold temperature that is small compared to the noise temperature of the set-up. Under that circumstance, the uncertainty is limited by uncertainty in the *ENR*, and measurement uncertainty of Y-factor. We reduce the uncertainty in Y-factor by selecting a source whose mismatch remains constant between switched states and remains low over the bandwidth of the determination.

As seen from the sample calculations, increasing the hot temperature does not produce as significant an effect as reducing the cold temperature of the noise source. However, we recommend to check whether the Boltzmann approximation holds at cryogenic temperature for the frequency range of interest. For example, Boltzmann approximation does not hold at liquid He temperature for frequency of 1 THz. At 1 THz, the zero-point power spectral density dominates at liquid He temperature for the noise source. At current state of the art the noise temperature of THz devices is north of 870 K so the Boltzmann approximation does hold for THz radiometers.

Computational Uncertainty Estimate (with Gain Fluctuation)

- $Y_{SUT} = \left(\frac{G_{H,SUT}}{G_{C,SUT}}\right)\left(\frac{T_{H,Src}^{SUT} + T_{SUT}}{T_{C,Src}^{SUT} + T_{SUT}}\right)$
- Denote $G_{C,SUT} = G$, $G_{H,SUT} = G + \Delta G$.
  - $Y_{SUT} + \Delta Y_{SUT} = \left(1 + \frac{\Delta G}{G}\right) Y_{SUT}$
  - $\frac{\Delta Y_{SUT}}{Y_{SUT}} = \frac{\Delta G}{G}$
- Suppose the source drives SUT gain to 1-dB compression @$T_{H,SUT}^{Source}$.
  - $\frac{\Delta G}{G} = -0.2$, and $\frac{\Delta T_{SUT}}{T_{SUT}} = 0.51$!

This slide describes the effect of Y-factor uncertainty caused by non-linearity, whether incidental or compressive. It points to a limit on *ENR*, which cannot be so high that it drives the radiometer into compression.

Computational Uncertainty Estimate (during Calibration)

- During calibration, same considerations apply for the NA as the SUT
  - $\frac{\Delta T_{NA}}{T_{NA}} = \sqrt{\left(1 + \frac{T_{C,Src}^{NA}}{T_{NA}}\right)^2 \left\{\left(\frac{\Delta ENR_{Src}^{NA}}{ENR_{Src}^{NA}}\right)^2 + \left[\ln\left(1 + \frac{\Delta Y_{NA}/Y_{NA}}{1 - 1/Y_{NA}}\right)\right]^2\right\} + \left(\frac{\Delta T_{C,Src}^{NA}}{T_{C,Src}^{NA}}\right)^2 \left(\frac{T_{C,Src}^{NA}}{T_{NA}}\right)^2}$
- In general, the NA noise input is at IF whereas the SUT noise input is at RF, so *ENR* and T_C are different for the noise source .
  - For $T_{C,Src}^{NA}$ = 300 K, $T_{H,Src}^{NA}$ = 6100 K, T_{NA} = 2610 K, Y_{NA} = 2.99, $\frac{\Delta ENR_{Src}^{NA}}{ENR_{Src}^{NA}} = \frac{\Delta T_{C,Src}^{NA}}{T_{C,Src}^{NA}} = \frac{\Delta Y_{NA}}{Y_{NA}} = 0.03$, ENR_{Src}^{NA} = 20, $\frac{\Delta T_{NA}}{T_{NA}}$ = 0.06.
- Requires care in design of noise source to ensure that mismatch is
  - Same for H and C states (therefore, cancels out in computing Y-factor)
  - Low so that loss contributes insignificant noise
- Test set-ups prevent interfering signals from entering the input

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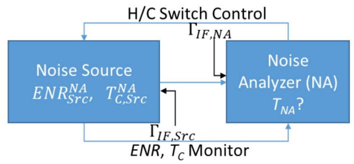
Here it is seen that the noise temperature of a typical NA is in the low thousands of K and does not require a cryogenic noise source. The main source of error in determining the noise temperature of the NA is mismatch. Mismatch that produces differences in transferred power between hot and cold states are particularly sensitive. Of course, interfering signals such as hum and RFI should be avoided.

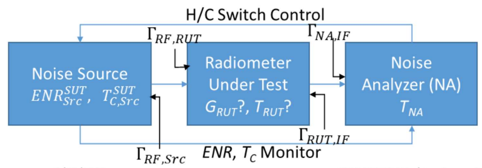
Y-Factor Test

Calibration



Measures	Computes
$P_{H,NA}$, $P_{C,NA}$	$Y_{NA} = P_{H,NA}/P_{C,NA}$ $T_{NA}/T_0 = ENR_{Src}^{NA}/(Y_{NA} - 1) - T_{C,Src}^{NA}/T_0$

Measurement of $SUT = RUT + NA$



Measures	Computes
$P_{H,SUT}$, $P_{C,SUT}$	$Y_{SUT} = P_{H,SUT}/P_{C,SUT}$ $T_{SUT}/T_0 = ENR_{Src}^{SUT}/(Y_{SUT} - 1) - T_{C,Src}^{SUT}/T_0$ $G_C = P_{C,SUT}/P_{C,NA}$ $G_{RUT} = G_C(Y_{SUT} - 1)/(Y_{NA} - 1)$ $T_{RUT} = T_{SUT} - T_{NA}/G_{RUT}$

Uncertainty Estimate (RUT Noise Temperature & Gain)

- $\delta T_{RUT} = \delta T_{SUT} - (\delta T_{NA}/T_{NA})(T_{NA}/G_{RUT}) + (\delta G_{RUT}/G_{RUT})(T_{NA}/G_{RUT})$
  $= T_{SUT}\frac{\delta T_{SUT}}{T_{SUT}} - (T_{SUT} - T_{RUT})\left(\frac{\delta T_{NA}}{T_{NA}} - \frac{\delta G_{RUT}}{G_{RUT}}\right)$
- $\frac{\delta T_{RUT}}{T_{RUT}} = \frac{T_{SUT}}{T_{RUT}}\frac{\delta T_{SUT}}{T_{SUT}} - \left(\frac{T_{SUT}}{T_{RUT}} - 1\right)\left(\frac{\delta T_{NA}}{T_{NA}} - \frac{\delta G_{RUT}}{G_{RUT}}\right)$
- $\frac{\Delta T_{RUT}}{T_{RUT}} = \sqrt{\left(\frac{T_{SUT}}{T_{RUT}}\right)^2\left(\frac{\Delta T_{SUT}}{T_{SUT}}\right)^2 + \left(\frac{T_{SUT}}{T_{RUT}} - 1\right)^2\left[\left(\frac{\Delta T_{NA}}{T_{NA}}\right)^2 + \left(\frac{\Delta G_{RUT}}{G_{RUT}}\right)^2\right]}$
- $\delta G_{RUT} = \delta G_C \frac{Y_{SUT}-1}{Y_{NA}-1} + G_C \frac{\delta(Y_{SUT}-1)}{Y_{NA}-1} - G_C \frac{Y_{SUT}-1}{Y_{NA}-1}\frac{\delta(Y_{NA}-1)}{Y_{NA}-1}$
  $= G_{RUT}\left(\frac{\delta G_C}{G_C} + \frac{\delta(Y_{SUT}-1)}{Y_{SUT}-1} - \frac{\delta(Y_{NA}-1)}{Y_{NA}-1}\right)$
  $= G_{RUT}\left(\frac{\delta G_C}{G_C} + \delta[\ln(Y_{SUT} - 1)] - \delta[\ln(Y_{NA} - 1)]\right)$
- $\left(\frac{\Delta G_{RUT}}{G_{RUT}}\right)^2 = \left(\frac{\Delta G_C}{G_C}\right)^2 + \left[\ln\left(1 + \frac{\Delta Y_{SUT}/Y_{SUT}}{1 - 1/Y_{SUT}}\right)\right]^2 + \left[\ln\left(1 + \frac{\Delta Y_{NA}/Y_{NA}}{1 - 1/Y_{NA}}\right)\right]^2$

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The next step is to determine uncertainty in determination of radiometer gain and noise temperature. Taking the differential of the expression for noise temperature and combining terms in RSS fashion, we see that contributions from uncertainties in noise analyzer temperature and radiometer gain can be diluted if T_{SUT} is close to T_{RUT}, which is to say that the second stage contribution to T_{SUT} is negligible. We will examine this quantitatively later.

For now, we see that contributions to uncertainty in G_{RUT} arise from the Y-factors and from the ratio, G_C, of powers in the cold condition in both configurations.

Measurement Uncertainty Estimate

- Arises during measurement of power: $P_{H,NA}$, $P_{C,NA}$, $P_{H,SUT}$, $P_{C,SUT}$
- Requires care in design of noise source to ensure that mismatch is
  - same for C and H states (therefore, cancels out in computing Y-factor)
  - low so that associated loss contributes insignificant noise
- Largest impact of mismatch is in estimation of $G_C = P_{C,SUT}/P_{C,NA}$, when measurement occurs for noise sources at different frequencies
- Behavior of test set-up is linear over C and H states of noise source
- Test set-ups must prevent interfering signals from entering the inputs

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Our encounter with G_C is the first direct instance where consideration of mismatch has no workaround mitigation and must be confronted head-on. This is because the participating powers are measured at different frequencies. Also, unlike previous quantities where the uncertainty was of computational origin, here the uncertainty is directly related to the measurement.

It may be thought that we may measure the complex scattering matrices of the noise source, radiometer, and noise analyzer with a network analyzer and use these to compute the power delivered to the radiometer input from the noise source, and power delivered to the NA from the radiometer output. However, uncertainty is related to the residual mismatch and it is increasingly difficult to make accurate phase measurement when the mismatch is small. This is because if the reflection from a unit is small, the amplitude of the reflected wave is low enough that it is overwhelmed by noise from the floor of the network analyzer. For this reason, it is usual practice to measure only the magnitude (or equivalently, VSWR) of units, and consider the two cases where each reflection reinforces the other and cancels the other.

Measurement Uncertainty (Mismatch)

- Uncertainty in measured power with RUT inserted
  - $P'_{C,SUT}\text{(max)} = \frac{P_{C,SUT}}{\left|1-|\Gamma_{RF,Src}||\Gamma_{RF,RUT}|\right|^2 \left|1-|\Gamma_{IF,RUT}||\Gamma_{IF,NA}|\right|^2}$
  - $P'_{C,SUT}\text{(min)} = \frac{P_{C,SUT}}{\left|1+|\Gamma_{RF,Src}||\Gamma_{RF,RUT}|\right|^2 \left|1+|\Gamma_{IF,RUT}||\Gamma_{IF,NA}|\right|^2}$
- Uncertainty in measured power with direct connection of source to NFA
  - $P'_{C,NA}\text{(max)} = \frac{P_{C,NA}}{\left|1-|\Gamma_{IF,Src}||\Gamma_{IF,NA}|\right|^2}$
  - $P'_{C,NA}\text{(min)} = \frac{P_{C,NA}}{\left|1+|\Gamma_{IF,Src}||\Gamma_{IF,NA}|\right|^2}$
  - $G'_C\text{(max)} = \frac{P'_{C,SUT}(max)}{P'_{C,NA}(min)} = G_C \frac{\left|1+|\Gamma_{IF,Src}||\Gamma_{IF,NA}|\right|^2}{\left|1-|\Gamma_{RF,Src}||\Gamma_{RF,RUT}|\right|^2 \left|1-|\Gamma_{IF,RUT}||\Gamma_{IF,NA}|\right|^2}$
  - $G'_C\text{(min)} = \frac{P'_{C,SUT}(min)}{P'_{C,NA}(max)} = G_C \frac{\left|1-|\Gamma_{IF,Src}||\Gamma_{IF,NA}|\right|^2}{\left|1+|\Gamma_{RF,Src}||\Gamma_{RF,RUT}|\right|^2 \left|1+|\Gamma_{IF,RUT}||\Gamma_{IF,NA}|\right|^2}$
  - $\frac{\Delta G_C}{G_C} = \frac{G'_C(max) - G'_C(min)}{G_C} = \frac{\left|1+|\Gamma_{IF,Src}||\Gamma_{IF,NA}|\right|^2}{\left|1-|\Gamma_{RF,Src}||\Gamma_{RF,RUT}|\right|^2 \left|1-|\Gamma_{IF,RUT}||\Gamma_{IF,NA}|\right|^2} - \frac{\left|1-|\Gamma_{IF,Src}||\Gamma_{IF,NA}|\right|^2}{\left|1+|\Gamma_{RF,Src}||\Gamma_{RF,RUT}|\right|^2 \left|1+|\Gamma_{IF,RUT}||\Gamma_{IF,NA}|\right|^2}$

In the above calculation, the negative sign corresponds to reflection cancellation whereas the positive sign corresponds to reflection reinforcement. This method enables us to estimate the maximum and minimum power delivered in each circumstance, and calculate the uncertainty in G_C.

Uncertainty Estimate (RUT Noise Temperature & Gain)

- $\delta T_{RUT} = \delta T_{SUT} - (\delta T_{NFA}/T_{NA})(T_{NA}/G_{RUT}) + (\delta G_{RUT}/G_{RUT})(T_{NA}/G_{RUT})$

  $= T_{SUT}\frac{\delta T_{SUT}}{T_{SUT}} - (T_{SUT} - T_{RUT})\left(\frac{\delta T_{NA}}{T_{NA}} - \frac{\delta G_{RUT}}{G_{RUT}}\right)$
- $\frac{\delta T_{RUT}}{T_{RUT}} = \frac{T_{SUT}}{T_{RUT}}\frac{\delta T_{SUT}}{T_{SUT}} - \left(\frac{T_{SUT}}{T_{RUT}} - 1\right)\left(\frac{\delta T_{NA}}{T_{NA}} - \frac{\delta G_{RUT}}{G_{RUT}}\right)$
- $\frac{\Delta T_{RUT}}{T_{RUT}} = \sqrt{\left(\frac{T_{SUT}}{T_{RUT}}\right)^2\left(\frac{\Delta T_{SUT}}{T_{SUT}}\right)^2 + \left(\frac{T_{SUT}}{T_{RUT}} - 1\right)^2\left[\left(\frac{\Delta T_{NA}}{T_{NA}}\right)^2 + \left(\frac{\Delta G_{RUT}}{G_{RUT}}\right)^2\right]}$
- $\delta G_{RUT} = \delta G_C\,\frac{Y_{SUT}-1}{Y_{NA}-1} + G_C\,\frac{\delta(Y_{SUT}-1)}{Y_{NA}-1} - G_C\,\frac{Y_{SUT}-1}{Y_{NA}-1}\frac{\delta(Y_{NA}-1)}{Y_{NA}-1}$

  $= G_{RUT}\left(\frac{\delta G_C}{G_C} + \frac{\delta(Y_{SUT}-1)}{Y_{SUT}-1} - \frac{\delta(Y_{NA}-1)}{Y_{NA}-1}\right)$

  $= G_{RUT}\left(\frac{\delta G_C}{G_C} + \delta[\ln(Y_{SUT} - 1)] - \delta[\ln(Y_{NA} - 1)]\right)$
- $\left(\frac{\Delta G_{RUT}}{G_{RUT}}\right)^2 = \left(\frac{\Delta G_C}{G_C}\right)^2 + \left[\ln\left(1 + \frac{\Delta Y_{SUT}/Y_{SUT}}{1 - 1/Y_{SUT}}\right)\right]^2 + \left[\ln\left(1 + \frac{\Delta Y_{NA}/Y_{NA}}{1 - 1/Y_{NA}}\right)\right]^2$

The uncertainty in G_C, permits calculation of uncertainty in gain, which in turn allows computation of uncertainty in the noise temperature of the radiometer. See Excel workbook.

Recommended Practices

- From $\frac{\Delta T_{RUT}}{T_{RUT}} = \sqrt{\left(\frac{T_{SUT}}{T_{RUT}}\right)^2 \left(\frac{\Delta T_{SUT}}{T_{SUT}}\right)^2 + \left(\frac{T_{SUT}}{T_{RUT}} - 1\right)^2 \left[\left(\frac{\Delta T_{NA}}{T_{NA}}\right)^2 + \left(\frac{\Delta G_{RUT}}{G_{RUT}}\right)^2\right]}$, it is evident that the smallest uncertainty is achieved if the second stage contribution to T_{SUT} is small, i.e., if $T_{NA} \ll G_{RUT} T_{RUT}$.
- Choose source with $T_{C,SUT}^{Source} \ll T_{SUT}$ to reduce the uncertainty in T_{SUT}.
- Switch noise source between H & C states rapidly enough to counter effect of gain variations and average Y-factors over multiple switching cycles to reduce uncertainty of estimate.
- To reduce uncertainty in gain, improve match of noise source to the best extent possible to mitigate
  - variation in source match between H & C states
  - uncertainty contribution due to source mismatch
    - Right-size bandwidth of source relative to RUT (use different sources for NA and SUT)

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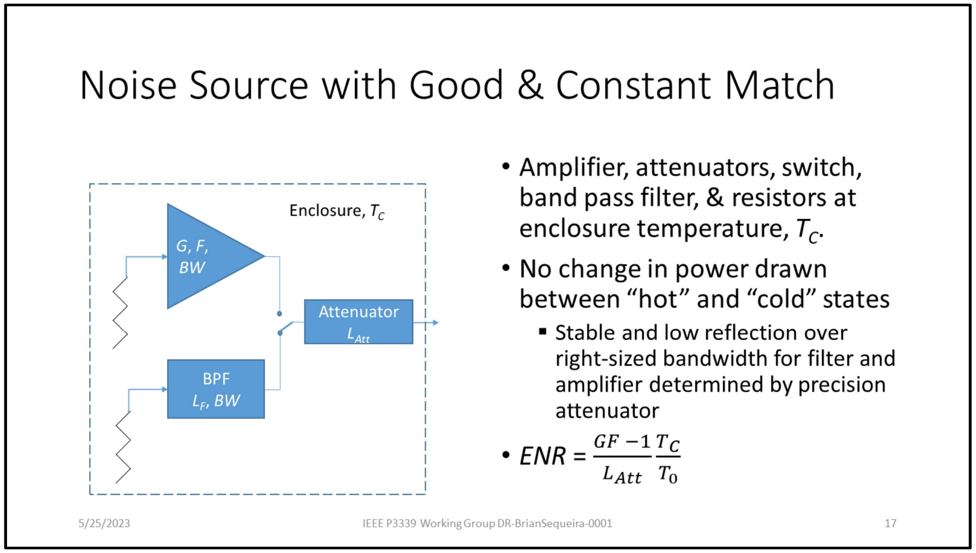
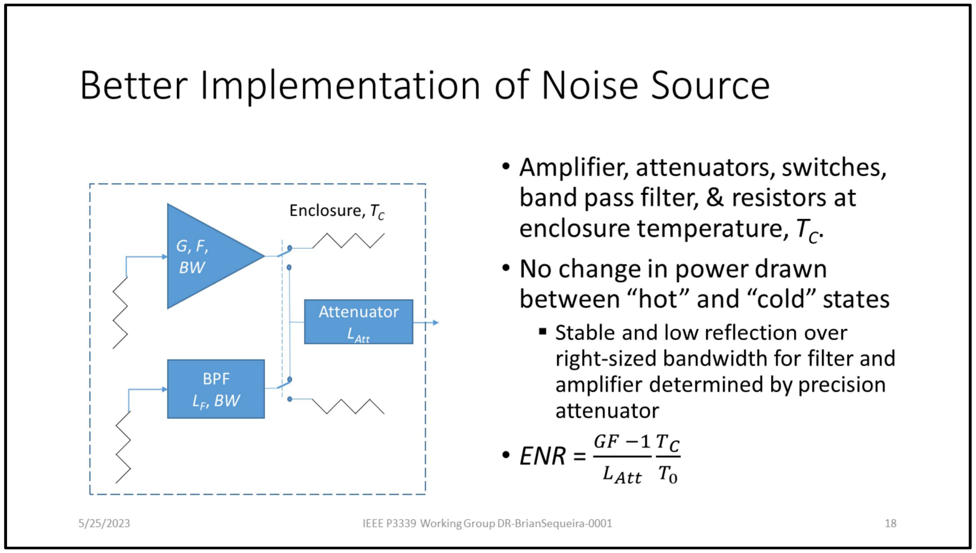
See Excel workbook. To obtain low uncertainty, ensure that the second-stage effect of the NA is insignificant by exploiting the high gain of the radiometer. In this case, the noise temperature of the radiometer is nearly the same but slightly less than the noise temperature of the radiometer and NA combined. The combined noise temperature is best determined under the condition that it is suitably greater than the cold temperature of the noise source. For very cold noise sources check the validity of Boltzmann's approximation to Planck's law and adjust for estimating the *ENR* accordingly.

Reduce the impact of gain fluctuations by switching sufficiently rapidly between hot and cold states of the noise source, and ensuring that the radiometer is operating linearly. Be especially aware of interfering signals that contaminate the radiometer's response.

Reduce the effects of impedance mismatch by designing the noise source for the best possible match that remains constant over switched states as well as offers low reflection over the bandwidth of the radiometer. These objectives are best achieved by choosing to match the source over a bandwidth that is only slightly larger than the radiometer bandwidth. It is recommended that separate noise sources be used at RF and IF. At RF, the percentage bandwidth over which the noise source must achieve good impedance match is smaller than at IF, and the *ENR* is smaller because of the need for linear operation with a high-gain radiometer. At RF, the need for cryogenic operation is greater than at IF for low-noise radiometers.



This slide presents a block diagram of a noise source that satisfies the criterion of a low and state-independent reflection coefficient, low cold temperature if needed, and optimum *ENR*. The enclosure is refrigerated to maintain constant temperature within which resistors, band-pass filter, amplifier, switch, and attenuator operate. This reduces temperature-related drift in the characteristics of all the components. An attenuator constructed out of precision resistive material provides low reflection coefficient at the output that is immeasurably constant for both switch positions if the attenuation is high enough and because the match of the amplifier and filter can be designed low enough over a right-sized bandwidth. The ENR of the source is determined by the gain noise-factor product of the amplifier, the attenuation of the precision attenuator, and the normalized temperature of the enclosure. This type of source can operate at cryogenic temperatures because of advances in device technology.



Here is a better implementation of the noise source which replaces the single-pole double-throw (SPDT) switch of the previous design with ganged double-pole, double-throw (DPDT) switches and two resistive terminations. This feature properly terminates the filter and amplifier when not connected to the output. One advantage is that the amplifier has substantially the same termination in both switch states and remains stable. Another advantage is that noise that propagates through the amplifier and filter is properly absorbed by terminations in all switch states. In the previous design, the unterminated device has noise reflected back from the open port of the switch, which creates random standing-wave patterns between the switch and that device. Practical switches have less than perfect isolation, and this random standing-wave noise couples to the output. This is especially the case at the amplifier, where its noise is comparably greater than noise that emerges from the filter. Also filters behave predictably when properly terminated at input and output.