Sampling For Radiometers

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Today we discuss considerations for sampling noise in a radiometer.

Radiometer Sampling

- Sampling in a single-channel radiometer is driven by
  - Digital image rejection (20 to 30 dB better than analog image rejection)
  - Rejection of intermodulation products
  - LO phase noise rejection
  - Suppression of jitter of the sampler itself
  - Suppression of spurious correlation inherent in the sampling process
- Choice of IF crucial to successful digital implementation of radiometer

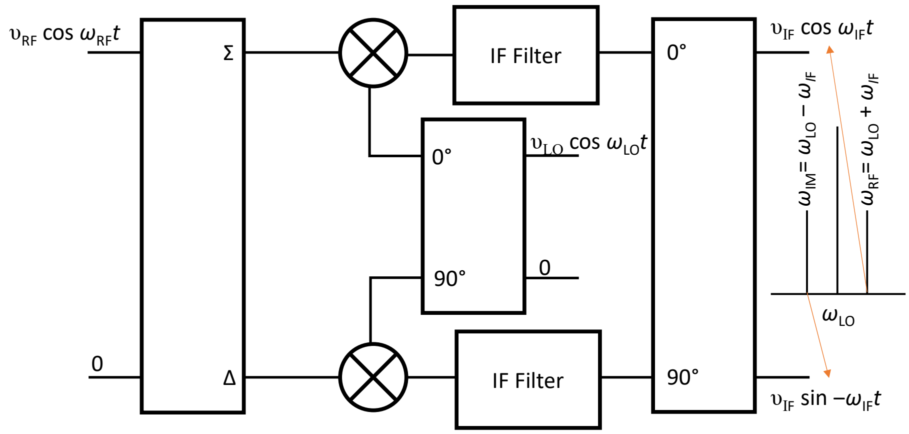
As we shall see shortly, implementation of sampling of noise must consider rejection of noise from sources other than the desired source. Examples of spurious sources of noise are image noise, intermodulation products, and phase noise from the local oscillator and sampling clock. Because our method uses correlation, sampling must also consider spurious correlation that could occur from the process.

Our preferred method implements as much of processes in the numerical (digital) domain, because numerically-implemented functions are immune from variations due to temperature and aging. Such variations are confined to the input data into the digitally implemented functions.

Digitally-implemented functions are also cleaner. For example, digital multipliers of two waveforms at frequencies f_1 and f_2 generate a spectrum at $f_1 \pm f_2$ only. Analog diode multipliers generate spectra at $mf_1 \pm nf_2$ where m and n are integers. A typical analog design entails a choice of frequencies of the individual waveforms such that only $f_1 - f_2$ falls within the desired band. Digitally-implemented filters produce repeatable characteristics independent of temperature and aging.

However, the desired signals or noise to be measured reside in the analog domain, and require careful selection of analog components to ensure that the digital implementation successfully accomplishes the objectives previously mentioned. In this regard, for a given bandwidth, the selection of center frequency of that band and associated sampling rate are particularly crucial.

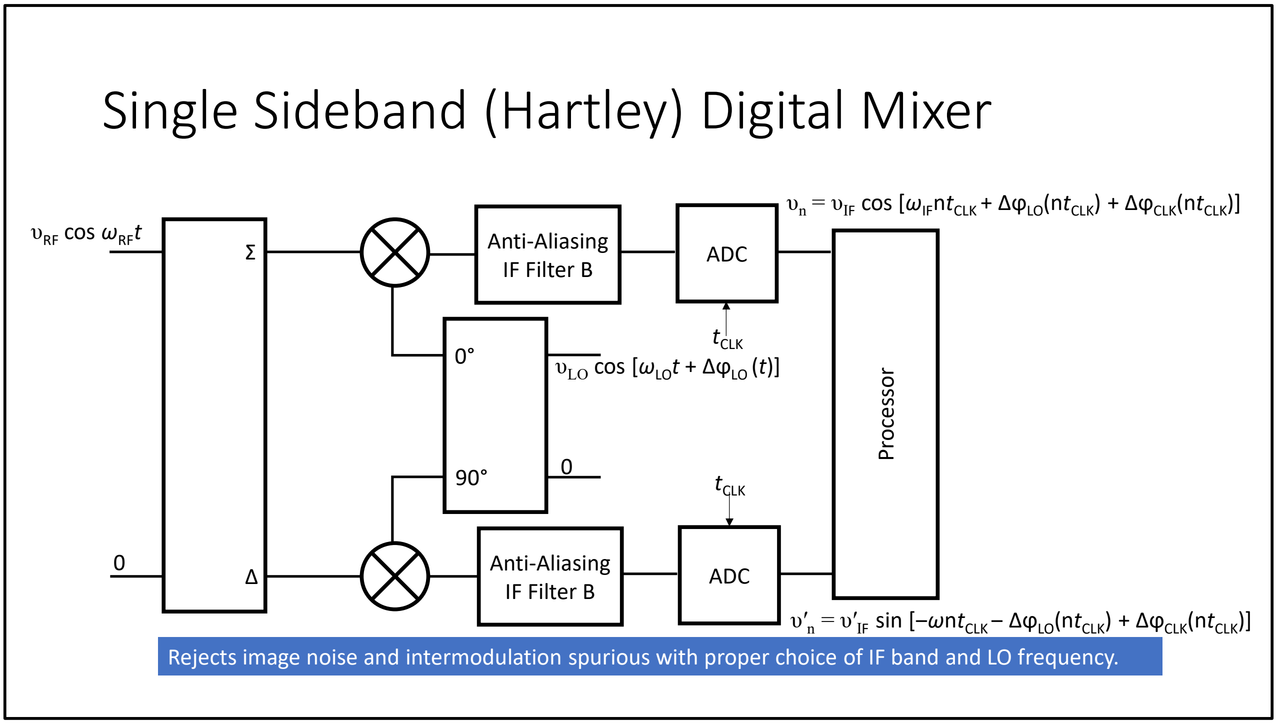
Ideal Single Sideband (Hartley) Mixer



Rejects image band and intermodulation spurious with proper choice of IF band and LO frequency

This diagram shows the functional form of an ideal image-reject mixer. An ideal 180° hybrid splits the incoming RF signal into equal-amplitude, equal-phase signals that drive the inputs of two identical mixers. An ideal local oscillator (LO) through an ideal 90° hybrid, drives corresponding ports of said mixers with equal amplitude, quadrature phase signals. The ideal mixers generate sum and difference frequencies of the RF and LO as well as intermodulation products of these. IF filters at the mixers output ports selectively allow only the difference frequency, also called the intermediate frequency (IF) to pass to the inputs of an ideal 90° hybrid combiner as shown.

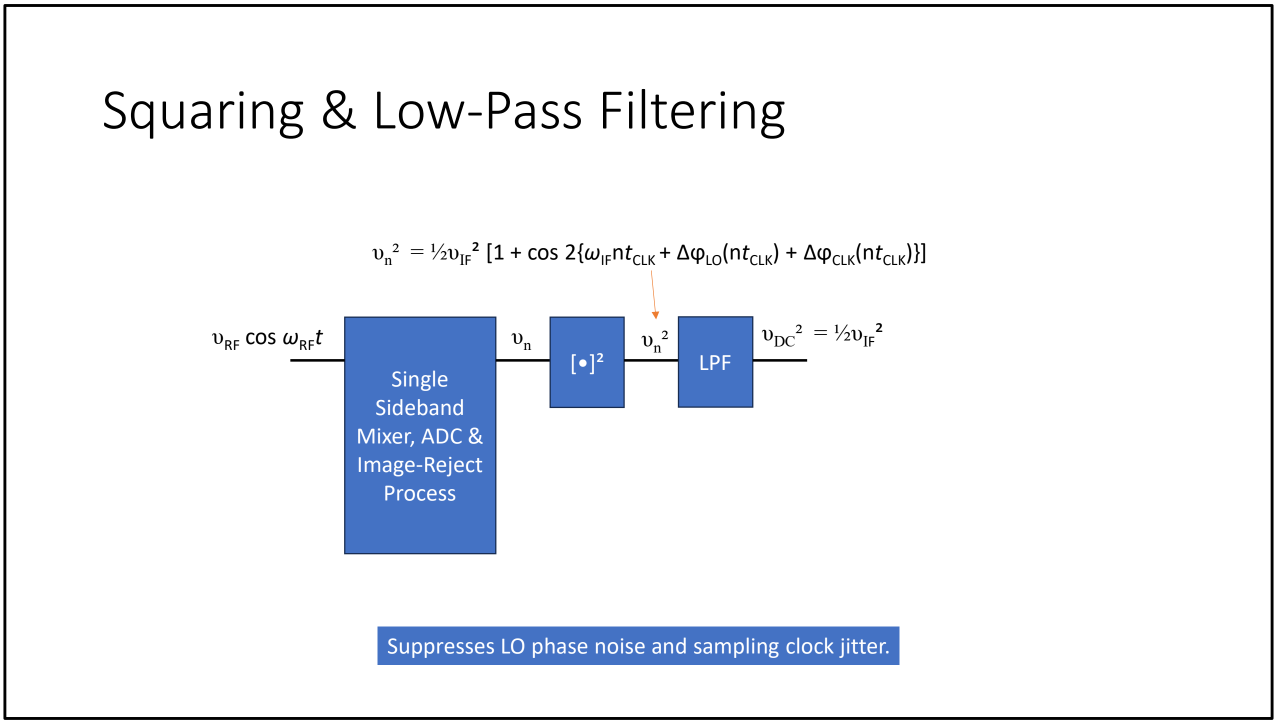
For a given LO frequency there are two RF frequencies: one above and one below the LO frequency, that yield the same IF. These RF frequencies are called images of each other. The quadrature combiner causes the image frequency to be rejected at one port and reinforced at the other port. Thus, by selecting the appropriate port, the image is rejected.



This diagram shows a digital implementation of the mixer in the previous slide. The components are not ideal: the outputs of the 180° hybrid do not have equal amplitude and phase, the outputs of the LO do not have equal amplitudes and are not in perfectly quadrature phase, the mixers are not identical, and the output combiner is likewise non-ideal. Furthermore, these amplitude and phase characteristics depend on frequency within the IF band.

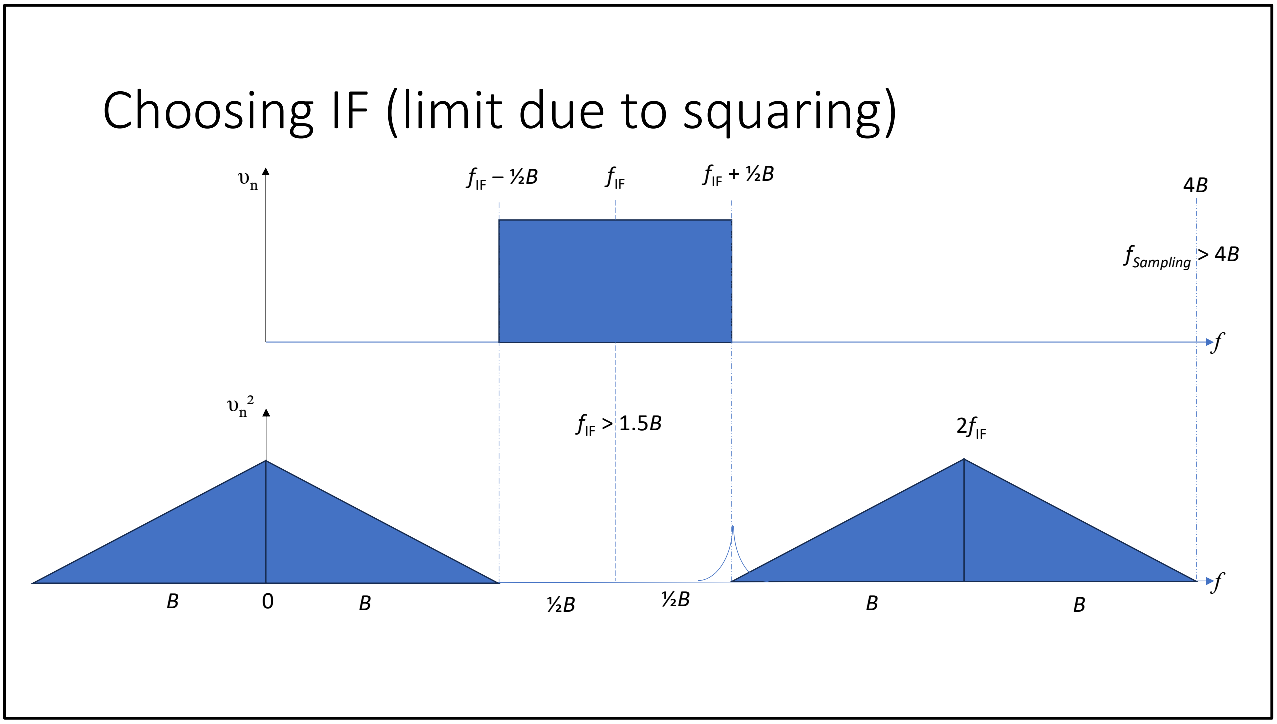
To handle these impairments, the IF filter of the previous slide is replaced by anti-aliasing filters whose outputs are digitized by analog-to-digital converters (ADCs). The digitized samples are stored in memory of a processor. A series of calibration tones are injected into the receiver at a given temperature. At each tone and temperature, the processor performed a FFT on the two digitized streams and determines the deviation from equal amplitude and departure from quadrature phase between them. It then uses these measurements to compute and store these as corrections to be applied. In thia way, the corrected streams provide superior image rejection compared to analog methods.

However, non-deal behavior brings impairments other than image rejection into the measurement. The LO is non-ideal, the mixers generate noise, and the sampling clock has jitter. Moreover, noise contributions due to phase noise of the LO and jitter from the sampling clock are multiplicative as shown in sample n represented in each sequence.



The processor suppresses multiplicative noise by squaring and filtering the samples as shown. Squaring separates the response into DC and IF terms. The low pass filter suppresses the passage of the IF term which is a twice the IF and contains the multiplicative noise.

Herein lies the considerations for proper choice of IF and sampling rate.

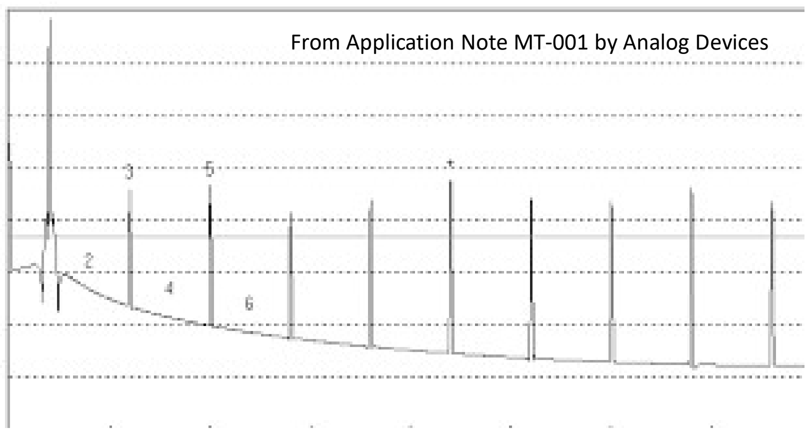


The top diagram is a frequency-domain plot of the IF band. Squaring entails multiplication and summing of every spectral component in the band with every other spectral component in the band. The largest number of multiplications results from multiplication of every spectral component with itself which produces a DC response and a second harmonic response which are therefore largest. The smallest number of multiplications occurs between spectral components $f_{IF} - ½B$ and $f_{IF} + ½B$, which therefore has the smallest response. Between these extremes are multiplications between spectral components at other frequency separations. The number of multiplications fall off linearly with frequency separation thus yielding the triangular distribution shown in the lower plot.

To prevent self-interference from squaring, the second harmonic of the lower band edge of the top plot must not fall within the band. This is satisfied if $2(f_{IF} - ½B) > f_{IF} + ½B$, or $f_{IF} > 1.5B$. Thus, $2f_{IF} > 3B$, and, to prevent the multiplicative noise from the second harmonic spectrum of the IF from aliasing to baseband, the sampling clock rate $f_{Sampling} > 4B$, as depicted in this slide.

It should be noted that the phase noise at the 2nd harmonic of the IF has twice the phase deviation of the original IF signal, and therefore sufficient allowance should be made to ensure that this does not contaminate the desired baseband.

Choosing IF (limit due to sub-harmonics)

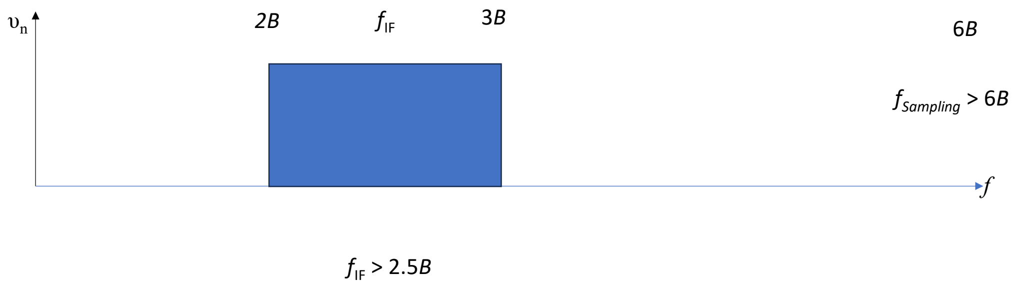


Samples at sub-harmonics of the sampling frequency strongly correlate and generate spurious responses at odd integer multiples of that sub-harmonic. Beware of low frequency interferers and noise.

Another effect occurs if a spectral component is a sub-harmonic of the sampling clock. In this case, a spurious correlation results that causes responses at odd integer multiples of the sub-harmonic. This effect compromises the spurious free dynamic range of the sampler. For this reason, it is essential that the anti-aliasing filters in slide 4 provide strong rejection of sub-harmonics of the sampling frequency both inside and outside the IF band. Thus, the filters cannot be low-pass filters.

Choosing IF to Avoid Sub-Harmonic Correlation

- $f_{IF} > 2.5B$
- $f_{Sampling} > 6B$



To avoid spurious correlation within the IF band of bandwidth B, $f_{IF} > 2.5B$, and $f_{Sampling} > 6B$, as indicated

Correlation Due to Oversampling

- Temporal response of a band-limited component with center frequency f_0:
  - $\chi = \exp(-\omega_0 t/Q)$
    - Quality factor $Q = f_0/B$, for bandwidth B.
    - Radian frequency $\omega_0 = 2\pi f_0$.
    - For $t = t_{Sampling} = 1/(6B)$, $\chi = \exp(-\pi/3) = 0.349$
    - At sample s_n, $\frac{s_{n-d}}{s_n} = \chi^d$

d	χ^d
1	0.349
2	0.122
3	0.043
4	0.015
5	0.005

The over-sampling by greater than a factor of 3 causes undesired correlation between samples because of the lingering response of the IF filter. As shown here, any sample, n, contains 35% of the amplitude response from sample n – 1, 12.2% of the response from sample n – 2, etc. This memory effect causes the digitized samples to be correlated even if the input analog samples are not. This spurious correlation may be avoided by suitable processing as described in the following slides.

Sampling Size

- For radiometers, typical sample size is large
  - For B = 100 MHz, $f_{Sampling} \geq 6B$ = 600 MS/s, in 1 ms #Samples = M = 600,000 (2^{19} = 524,288), so $\chi^M \approx 0$ (well below the underflow threshold of computers).

First, it is significant that typical radiometer operation uses large sample sizes. For the threshold sampling rates developed here the number of samples exceeds half a million. This fact simplifies suppression of this correlation.

Correlation Coefficient

- Digitized samples are correlated even if input to the digitizer is uncorrelated.
- Digitization preserves mean of the input if input is uncorrelated.
- Under condition of arbitrary sample size (not necessarily large)
- $\rho = \frac{\mathbf{COV}(s_{\mathrm{i+d}}, s_{\mathrm{i}})}{\sqrt{\mathbf{VAR}(s_{\mathrm{i+d}})\mathbf{VAR}(s_{\mathrm{i}})}} = \chi^{\mathrm{d}} \sqrt{\frac{1 - \frac{\chi^2}{1-\chi^2}\frac{1-\chi^{2\mathrm{M}}}{\mathrm{M}-1}}{1 - \frac{\chi^2}{1-\chi^2}\frac{1-\chi^{2(\mathrm{M-d})}}{\mathrm{M-d}-1}}}$
- For large M, small d, $\rho = \chi^{\mathrm{d}}$
- So staggered decimation mitigates sample-to-sample correlation.

As mentioned previously, the memory effect of the IF filter causes the digitized sequence of samples to be correlated even if the input analog sequence is not. However, if the analog sequence is uncorrelated and the sample size is large, digitization preserves the mean of the analog input. In radiometers the necessary bandpass characteristic of the anti-aliasing filter results in zero-mean input to the sampler. The digitized samples also have zero mean. The correlation coefficient of digitized samples of arbitrary size is shown here. For large sample size, the coefficient is related to a decimation depth.

Algorithm for Correlation-Suppressed Squaring

- Partition sample size into decimated sequences
  - Decimation must satisfy Nyquist criterion, e.g., for I-Q sampling @ 6*B* (d < 6), decimation d = 5, reduces effective sampling rate to 1.2*B* for I & Q.
  - Perform squaring & LPF operations with:
    - Sequence 1 using samples 1, 6, 11, 16, …
    - Sequence 2 using samples 2, 7, 12, 17, …
    - Sequence 3 using samples 3, 8, 13, 18. …
    - Sequence 4 using samples 4, 9, 14, 19, …
    - Sequence 5 using samples 5, 10, 15, 20, …
- Combine outputs of squaring from all decimated sequences

The decimation depth is necessarily small because Nyquist criterion must still be preserved after decimation. Thus, for I-Q sampling at 6*B*, the decimation depth cannot exceed 5. In this example, it suggests splitting the original sampled sequence into 5 decimated sequences for which the squaring and filtering operations are performed separately. The correlation between adjacent samples in the decimated sequence is reduced from 35% to 0.5%. The separate decimated correlations are combined to provide the total output. No samples are discarded.

Summary

- Preferred radiometer operation is in the numerical (digital) domain
  - Requires careful attention to selection of IF and sampling rate to
    - Suppress image response
    - Advantageously use squaring to eliminate multiplicative (phase) noise from LO & sampler
    - Avoid spurious correlations from sub-harmonics of the sampling rate
  - Requires staggered decimation algorithm to
    - Suppress spurious correlation from the consequent over-sampling process

The key takeaway from this presentation is that to obtain a preferred digital implementation of a radiometer, careful attention must be paid to the IF filter and sampling rate to suppress image response, eliminate multiplicative noise, and avoid spurious correlations from sub-harmonics of the sampling rate.

The resulting over-sampling from these considerations causes inherent correlation among digitized samples. Staggered decimation mitigates the effect of this correlation.